10 - Implication

Implication ("if p then q" or "$p \Rightarrow q$" or "p implies q") models valid deductions:

So all instances of this is true:

But:

Truth table for =>:

p	q	
T	T	
T	F	
F	T	
F	F	

Example 1.
"If pigs could fly, then I can swim."

"If 6 divides 4686, then 3 divides 4686."

Example 2. Job promises that "If you show up to work Monday, then you get the job."
But this turned out to be false. What can you conclude?

Example 3. Show $\neg(p \Rightarrow q) \equiv p \wedge \neg q$

p	q		
T	T		
T	F		
F	T		
F	F		

p	q		
T	T		
T	F		
F	T		
F	F		

Example 4. Negate "If my car broke, then I cannot come."

More translations of $p \Rightarrow q$:
"p only if q", "q whenever p", "q is necessary for p", "p is sufficient for q"

Order of operations:

Highest			Lowest

and read from left to right

Example 5. Fully parenthesize the following propositions.

$p \vee q \Leftrightarrow q$

$p \Rightarrow q \Rightarrow r \wedge s$

$p \Rightarrow q \Rightarrow r$

$\neg p \Leftrightarrow p \Leftrightarrow \neg(p \Leftrightarrow p)$

Definition.

Example 6. "If my car broke, then I cannot come."

Converse:

Contrapositive:

Exercise 3b shows the contrapositive is equivalent to the original implication.

Optional Homework due March 25th or 26th.
Show your work. Answer without work receives no credit.

1. Write "Stop, or I'll shoot" as an implication.
2. Negate the if-then statement: "If Sara lives in Athens, then she lives in Greece."
3. Show (a) $p \Rightarrow q \equiv \neg p \vee q$, (b) $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$, (c) $\big((p \Rightarrow q) \Rightarrow q\big) \equiv p \vee q$

Non-Homework Problems.
4. Translate "Children and seniors pay half price" and "Children or seniors pay half price" into logically equivalent statements.